

The Calculus of Change: Winning the Race to Green

Activity Overview

In this activity, students mimic professionals who must take only a few data points and create a mathematical picture to help make critical decisions. Students become a Pit Crew Chief who must use data to decide when and if to re-engineer their car mid-race. They begin to answer the monumental question of our day: How do we move to Green Transportation? In both scenarios, students acquire first hand experience in developing mathematics as a descriptive language.

The activity can serve as a first introduction or extension to linear equations, 2nd order equations (quadratics) and the concept of rates of change (derivatives) and areas under curves (integrals). First, students graph real data (discrete points), and look at means of describing the data with a trend line (linear regression, linear equations). Second, students look at the discrete differences of two lines then the accumulated difference (area between the lines, integral, quadratic function). Students make qualitative statements about the implications of these functions. In this example the functions describe lag time behind the leader in a race and total number of cars on the road. Finally, students use a slider to control the parameters of a function, and watch the shape of the function change as well as its transformation along the X axis.

Concepts

- Scatter plot of discrete data
- Linear regressions and equations, Quadratic equations
- Area formed by two equations between two limits (X values), integrals
- Growth rates, rates of change, derivatives • Curve fitting
- Understanding the narrative of discrete and continuous functions

TI-Nspire™ Applications

TI-Nspire, Graphs & Algebraic Functions, Lists & Spreadsheet, Notes, Data & Statistics

Teacher Preparation

This activity is designed for use in a Calculus classroom but can be used in an introductory Algebra classroom by omitting discussion of integrals and examining only the concept of area under the curve and increasing/decreasing functions in place of rates of change. It can be used to introduce the idea of trend lines to represent samples, discrete functions of samples, the real meaning of integrals, area under the curve and differences in areas (continuous functions), curve fitting.

- Prior to beginning, students should understand the idea of a trend line, have experience in linear regressions and/or writing linear equations and a basic understanding of area and areas between lines.
- Review the screenshots on pages 2 and 3 that demonstrate expected student results.
- **Download Activity Resources at www.Ten80Education.com/EdResources.html**
 1. TI-Nspire (.tns) Files 1, 2 and 3
 2. PPT/PDF Presentation of full activity

FastTrack RC Teams can extend the concept of area under the curve with the Pi Note: Calculating Acceleration Rates.

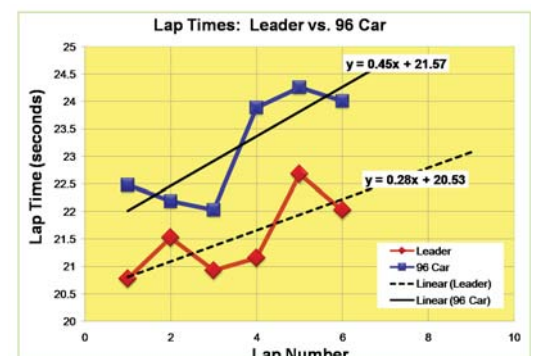
Classroom Management

- This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.
- For the most part, students will perform simple calculations and manipulate existing functions in the .tns files provided. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.
- The ideas contained in the following pages are intended to provide a framework as to how the activity will progress.

Table 1: Raw Race Data

Lap #	Leader Lap Time sec.	96 Car Lap Time sec.	Δ Lap Time sec.	Lag Behind Leader sec.
1	20.78	22.48	1.7	1.7
2	21.53	22.19	0.66	2.36
3	20.93	22.03	1.1	3.46
4	21.16	23.89	2.73	6.19
5	22.68	24.26	1.58	7.77
6	22.03	24.01	1.98	9.75
7				

Fig 1: X-Y Scatter & Trend Lines



Problem Part 1 – Short Pitting (TNS File 1)

Linear Equations, Discrete & Continuous Functions, Area Under Curve

In Part 1, you are the Crew Chief of DLP Technology's #96 Race Team. You're initially behind in the race as shown by data in Table 1. Your goal is to figure out when your car will be lapped so you can 'Short Pit' before then. After the unscheduled pit (a short pit), figure out if and when the 96 car will take the lead.

Step 1: (*spreadsheet or TI-Nspire*) Table 1 shows data a Crew Chief might receive during a race. Create an X-Y scatter plot for Lap Time as a function of Lap Number for the Leader and 96 as in Fig 1. Add trend lines for both teams using a linear regression. Alternatively, do it manually using $(Y = mX + b)$.

Discuss what the linear constants mean. Slope is the decay rate (how much the cars slow down each lap due to tire wear) and Y-intercept sets the 1st lap time. Together, the 1st lap time and decay rate dictate subsequent lap times.

Step 2: Open .tns File 1 showing both trend lines and the area beneath both lines. Discuss the meaning of these areas.

Sir Isaac Newton found the area under a curve by adding the area of trapezoids that fit under the curve, using ever smaller widths so that $(X_2 - X_1)$ approaches 0. Mathematically, taking the integral of a function is far easier! The area under each curve is the accumulated (total) drive time for that car between laps X_1 and X_2 . The lines show discrete lap times. The areas show total drive times.

Move the boundaries along the X-axis to show the areas increase and decrease. Move the trend lines (Y-intercept) up and down to represent higher lap times. Rotate the lines to change the slope. Change in slope represents a change in decay rate in lap times. The corresponding change in area represents a change in the total drive time for each car during the laps X_1 through X_2 .

Step 3: Lag time between drivers is the difference in their total drive times (how far behind). Lag time equals the difference between the two areas (with X values aligned). Lag Time = $DA_{96} - DA_{Lead} = \text{area between lines (dark area in Fig 3)}$.

Open File 1, Sheet 2. Added to your model is a function that represents the area between the curves, or lag time. To derive it, subtract the Leader trend line (f11) from 96 trend line (f12) and take the integral of the resulting linear equation. The integral of a linear equation is a quadratic equation as shown in Fig 3 as f1.

When the lag time (f1) = Leader's lap time (f11), the 96 will be lapped. You must Short Pit before then. Move the X-axis boundaries to see the intersection around lap 11.

Step 4: Open File 1, Sheet 4. The Crew Chief yells *PIT NOW!* in lap 9 when 15 seconds back. Say the 96 pits then returns on Leader lap 10, now 35 seconds back. Re-entry is represented by a point at the top center (~10, 35) in Fig 4.

By pitting before scheduled and making proper adjustments (springs, tire pressure, etc.), the 96 begins to gain on the Leader. New function f1 shows the new 96 trend line and old f1 does not apply after pitting. This new 96 trend line shows lower lap times and a slightly better decay rate than the Leader (slopes are close, but f1 slope is smaller than f11).

The area between lines f11 and f1 (with $X_1 = \text{Leader lap 10}$) now show the difference in drive times after the pit. To pass the Leader, the 96 has to gain the 35 seconds he's back upon re-entering the field.

Step 5: Open File 1, Sheet 5 to show f2, the post-pit lag time. Can 96 take lead before the Leader is scheduled to pit on lap 25? F2 shown in Fig 5 is derived just like the previous lag time: Subtract $(f11 - f1)$ and integrate the result. Define the constant in this function in 1 of 2 ways: (a) Add it to the TI-Nspire graph then move it to intersect with the re-entry point (10,35) or (b) Use this re-entry point to solve for the constant algebraically.

With $X_1=10$, the 96 will pass the Leader when (a) $DA_{Lead} - DA_{96} > 35$ or (b) $f2 = 0$. When will this happen?

What is the highest decay rate that will allow the 96 to pass by lap 25? What's the highest lap time?

Fig 2 (Sheet1): Lines & Area Under Curves

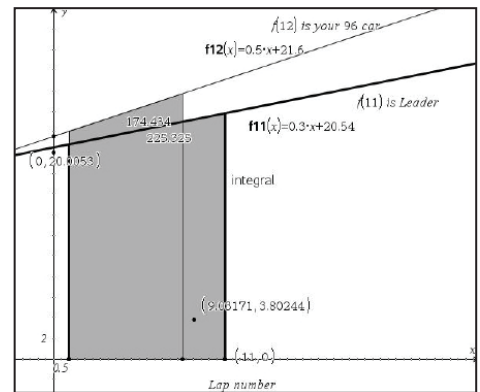


Fig 3 (Sheet2): Lag Time, Area Between Curves

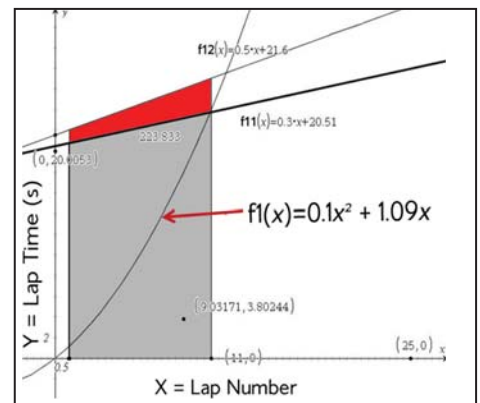


Fig 4 (Sheet4): Post-Pit Performance

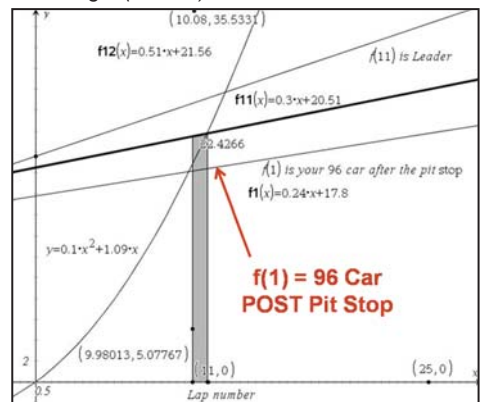
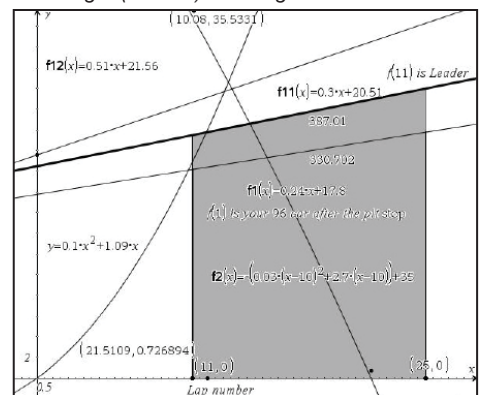


Fig 5 (Sheet5): Gaining on the Leader



Problem Part 2 – P.I.T. NOW!

Petroleum Independent Transportation (P.I.T.): Discrete & Continuous

As in Part 1, you are challenged to build a coherent, decision-making picture from a relatively few data points. This time, it is to tackle the monumental issue of how to change the US fleet of conventional cars to 'Green' cars.

Only continuous functions can give a living picture and narrative of how change happens (or doesn't!). Students will manipulate one such function describing the US car population then work with a part of that function to see how to use the TI-Nspire to make such continuous functions from discrete points.

Step 1: Open .tns File 2, Sheet 1 to reveal the curve shown in Fig 6. X-axis is the age of cars on the road today and the Y-axis is the number of cars that age. In another way of looking at it, X = year working backwards from 2008 (X=0 is 2008, X=20 is 1988) and the Y-axis is the number of cars put into service that year that are still on the road. Move X₂ to 25 to see how many cars still on the road today went into service in 1983. 2008?

What does the area here show? Just like in Part 1, the area under the curve is the integral of the continuous function. It represents the total number of working cars on the road today (US car population). Make X₁=0 and X₂=max to show the area under the curve = total car population in 2008 = ~250 million. Make X₂=9.5 to show that there are ~125 million cars up to 9.5 years old. That is known as the half-life of cars because half of all cars on the road are under 9.5 years old.

Step 2: Open File 2, Sheet 3. It is projected that Americans will only buy 10 million cars in 2009, even fewer cars through 2012 with sales beginning to recover in the following years. Make X₂=max to show total population in 2012. Make X₂ = 11 to show that the new half-life of cars will be 11 years under this scenario. The US government finds this troubling and is using the 2009 Stimulus Bill to incentivize people to retire cars 9 years and older.

Another policy goal is to put 1 million electric cars on the road by 2012. A triangle with an area equal to 1 MM is shown in the lower left corner. This 1 MM cars is a tiny 'drop in the bucket' compared to the total US fleet. What may a better goal for 2012 or 2020? We'll ask if your idea is realistic later in this activity.

Step 3: In the Short Pit activity, students fit the lag-time function (f2) to a known point. The simple quadratic format made that pretty easy to do. With data points on the car population projection, it is more complicated. Open File 2, Sheet 4 to illustrate curve fitting with a typical Guassian curve (logarithmic decay).

1. The first step in creating a continuous function is to enter the discrete points and see the pattern they create.
2. Next, choose a type of function that matches the pattern. Fig 7 looks like a Gaussian curve combined with an exponential decay (as shown in the formula). In this simplified example, you're working only with the Gaussian curve.
3. Finally, try different constants to fit the graph to the points. The TI-nspire provides sliders that allow effortless variation in each constant. What happens when you modify V4? V5? V6? Finally, try 0.21, 8.25 and 6.25.

Step 4: Now you know how to take discrete points, create a continuous function and play the "What-If Game". To effect real change, wouldn't 35 million cars by 2012 be better (what % would that be)?. What was your goal from Step 2?

Open .tns File 3 to reveal a growth curve. X = years from 2008 looking forward. Y = the number of electric cars put onto the road that year. In this image, the area under the curve is the total number of electric cars. The growth rate is the slope of the tangent to that curve. To hit that more ambitious goal, what do we need to be producing today? Next year? What rate of change is that? Is it a realistic goal? What infrastructure would be required to support that many electric cars?

The goal is not to answer these questions today. It is to pose the questions and see that it will take rigorous mathematical analysis like this to understand the situation we're in and find realistic, workable answers.

Fig 6 (File2/Sht1): Age of US Cars in 2008

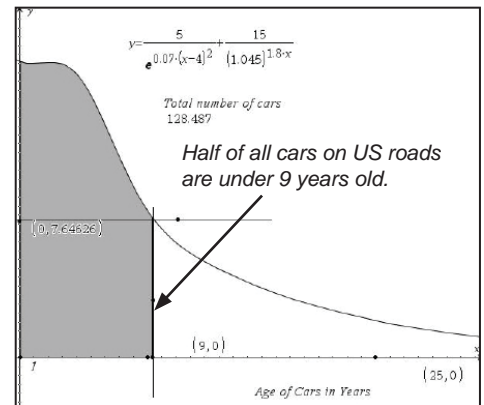


Fig 7 (File2/Sht3): Car Pop Projections to 2012

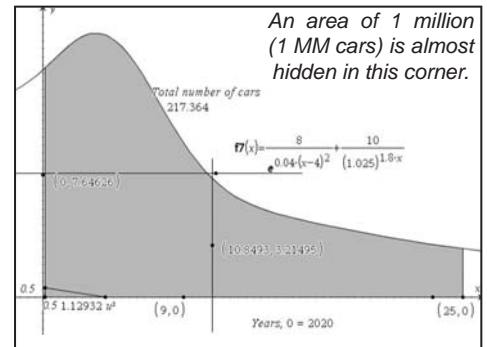


Fig 8 (File2/Sht4): Make a Continuous Function

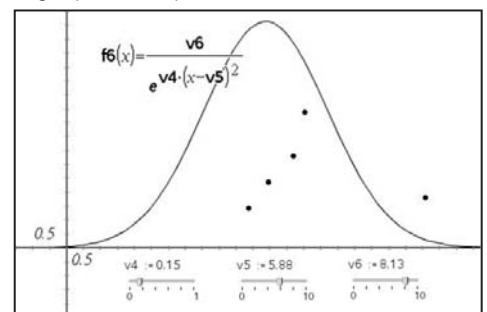


Fig 9 (File3): Growth Curve of E-cars, 2008+

